

Development of A New Flux Splitting Scheme

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Maximizing both accuracy and efficiency has been the primary objective in designing a numerical algorithm for computational fluid dynamics (CFD). This is especially important for solution of complex 3D problems which often involve Navier-Stokes equations with turbulence modeling and chemical species equations. Upwind schemes have been well received for both their capability of resolving discontinuities and their sound theoretical basis in characteristic theory for hyperbolic systems. Several flux splitting schemes, notably by Steger-Warming, Van Leer, Osher, and Roe, have been tested and discussed extensively in the past decade.

However, several inherent shortcomings exist in each of these schemes. For example, while the Van Leer scheme is simple, taking only $O(n)$ operations, n being the number of equations, and yields accurate solutions for inviscid problems, it suffers inaccuracy for predicting velocity and temperature fields in the viscous problems. The Roe scheme, commonly accepted as the most accurate scheme available currently, however is a great deal more complex and costly due to the matrix operation, requiring $O(n^2)$ operations. Moreover, the extension to the chemically reacting flows renders no unique way of defining the 'Roe-averaged' states. The Osher scheme has the smooth property and has recently been generalized by Suresh and Liou to deal with chemically reacting flows. But the determination of the intermediate states also requires $O(n^2)$ operations. Thus it is logical to ask whether there is room in the universe of upwind schemes for improvement to arrive at a simple ($O(n)$ operations) and accurate scheme for a wide range of problems.

In this paper, we summarize recent successes of a new splitting scheme for some model aerodynamic problems where Van Leer and Roe schemes failed. The new scheme is based on a rather different idea of splitting in which the convective and pressure terms are separated and treated differently in accordance with the underlying physical intuitions. We propose an appropriately defined cell-face advection Mach number using values from the two straddling cells via associated characteristic speeds. This interface Mach number is then used to determine the upwind extrapolation for the convective quantities. Next the pressure splitting is weighted using polynomial expansions of the characteristic speeds.

Thus, the name of the present scheme is properly coined as Advection Upstream Splitting Method (AUSM). The scheme is remarkably simple and yet its accuracy in the present study rivals and in some cases surpasses the Roe scheme in the Euler and Navier-Stokes solutions at considerably reduced computational effort. The detailed formulation of the scheme is not shown here due to space limitation. However it will appear elsewhere.

The calculation of the hypersonic conical flow demonstrates the accuracy of the splittings in resolving the flow in the presence of strong gradients. The temperature and pressure profiles of the first order results are shown in Figs. 1 (a) and (b). The Van Leer splitting is seen to produce a thicker boundary layer which in turn further displaces the shock wave. Both AUSM and Roe solutions are in excellent agreement.

The second series of tests involve the 2D inviscid flow over a NACA 0012 airfoil. The results, not included here, demonstrate that the level of entropy generation at the stagnation point is about three times smaller than the Roe solution.

In the third case we calculate a series of supersonic flows over a circular cylinder. The Roe splitting in all conditions and grids tested yields anomalous solutions (sometimes referred to as the carbuncle phenomenon), which may appear as nonsymmetric, protuberant, or indented contours, see Fig. 2. The mode of these non-physical solutions appears to be sensitive to changes in Mach number or grid. The AUSM, however, gives expected solutions in all calculations.

The fourth test deals with a 2D shock wave/laminar boundary-layer interaction. In Fig. 3, the AUSM is seen to give excellent agreement with the data especially in the reattachment region, which has been in defiance of many previous calculations in the literature. Also the oblique shock appears to be more tightly captured by the AUSM.

In summary, it appears that the new splitting scheme, AUSM, has delivered the promise by improving the accuracy as well as efficiency significantly. As usual, the final judgment will be decided via many more tests and further modifications of the scheme.

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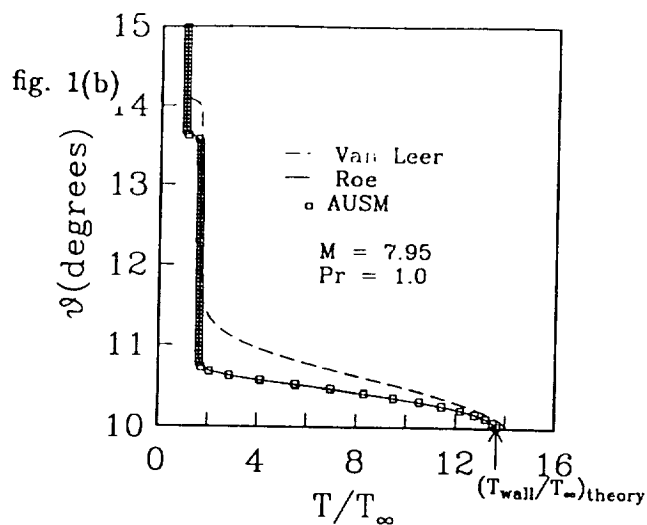
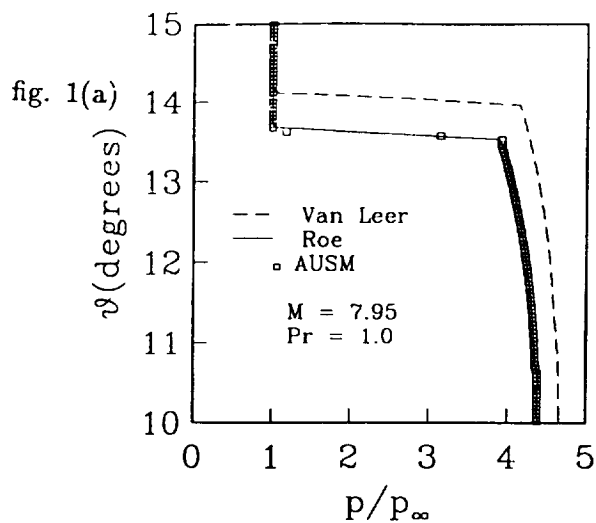


Fig. 1 Conic flow with $M_\infty = 7.95$, $Re = 4.2 \times 10^5$, and half cone angle = 10° : (a) pressure, and (b) temperature distributions.

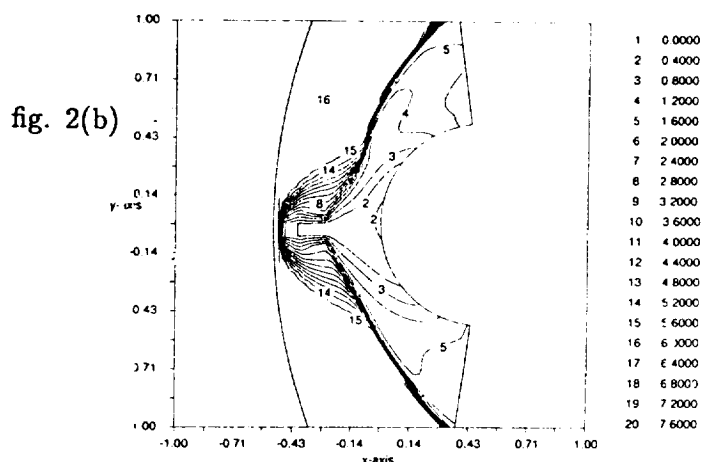
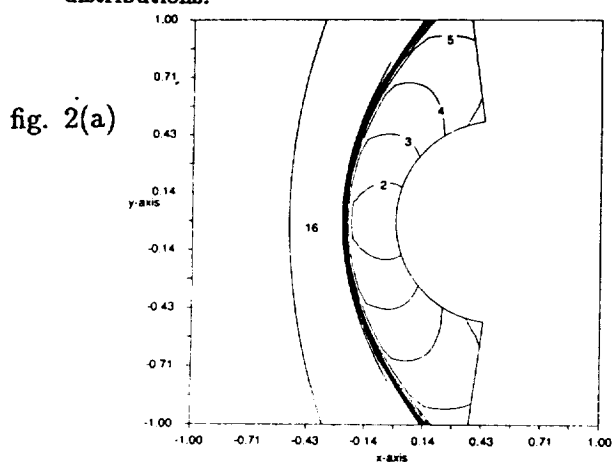


Fig. 2 Mach contours for a Mach 6 inviscid flow over a circular cylinder: (a) the AUSM solution, (b) the Roe solution displaying a protuberrant, two-shock contours.

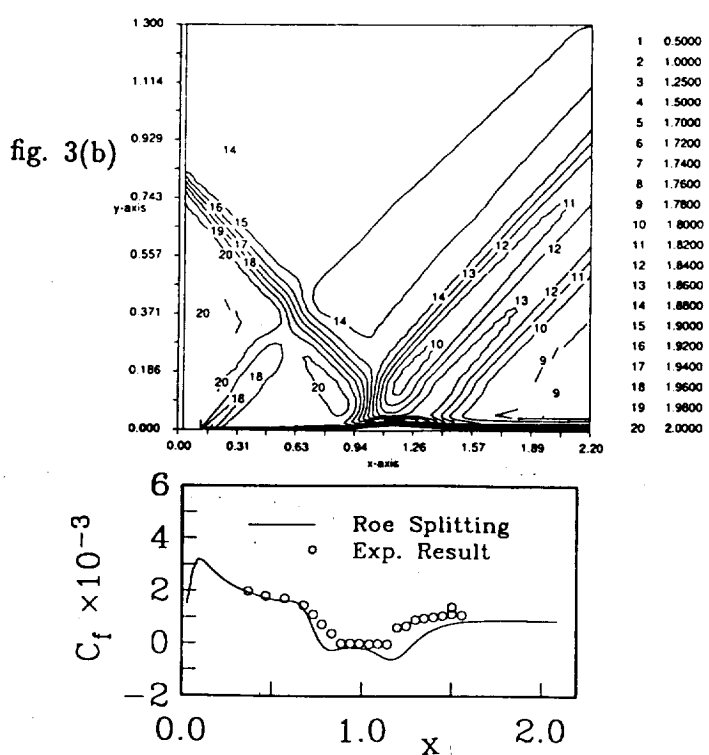
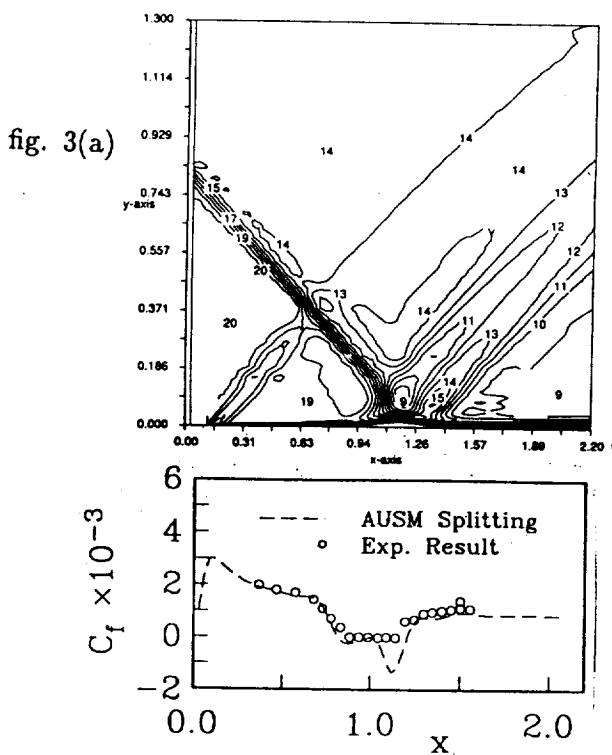


Fig. 3 Mach contours and comparison of skin friction with data for $M_\infty = 2.0$, $Re = 2.96 \times 10^5$ [Hakkinen et al]: (a) the AUSM solution, and (b) the Roe solution.